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# Why Must Incompatibility Be Symmetric?

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## Abstract

Why must incompatibility be symmetric? An odd question, but recent work in the semantics of non-classical logic, which appeals to the notion of incompatibility as a primitive and defines negation in terms of it, has brought this question to the fore. Francesco Berto proposes such a semantics for negation argues that, since incompatibility must be symmetric, double negation introduction must be a law of negation. However, he offers no argument for the claim that incompatibility really *must* be symmetric. Here, I provide such an argument, showing that, insofar as we think of incompatibility in normative pragmatic terms, it can play its basic pragmatic function only if it is symmetric. The upshot is that we can vindicate Berto's claim about the symmetry of incompatibility but only if we, *pace* Berto, think about incompatibility, in the first instance, as a *pragmatic* relation between *acts* rather than a *semantic* relation between *contents*.

**Keywords:** Incompatibility, Negation, Non-Classical Logic, Australian Plan, Transcendental Argument

## 1 Introduction

A car's being red is incompatible with its being blue, and, just as well, a car's being blue is incompatible with its being red. Someone's missing the bus is incompatible with their catching it, and, just as well, someone's catching the bus is incompatible with their missing it. Here we have facts consisting in a relation of incompatibility between two states of affairs or event types  $a$  and  $b$ , and, in both cases, where  $a$  is incompatible with  $b$ ,  $b$  is incompatible with  $a$ . So, incompatibility, at least in these cases, is symmetric. But does the concept of incompatibility deployed here, as such,

mandate symmetry? Can we rule out the possibility of there being contents, presumably contents quite different than something's being red or someone's missing the bus, that stand in asymmetric incompatibility relations? Until recently, this question had not received much philosophical attention. However, a new trend in thinking about negation in terms of incompatibility (Dunn 1993, 1996; Restall 1999, 2000; Berto 2015; Berto and Restall 2019) has brought it to the fore. A result due to Restall (2000) shows that, if we think of negation as defined on the basis of incompatibility, then double negation introduction ( $p \vDash \neg\neg p$ ) is a law of negation just in case incompatibility is symmetric. Berto (2015) argues that, since incompatibility must be symmetric, anything that is a negation operator must satisfy double negation introduction. However, he gives no argument for the claim that incompatibility *must* be symmetric. Neither he nor anyone else in this trend has said anything to positively rule out the possibility of asymmetrically incompatible contents. Here, I will show that we can indeed rule out such contents, but only if we think about incompatibility quite differently than it has been presumed we ought to think about it.

Drawing from Brandom (1994), I propose that we think of incompatibility, in the first instance, not as an alethic modal relation that obtains between worldly contents such as states of affairs, but a normative relation that obtains between discursive acts, fundamentally, between acts of making claims. Specifically, two claims  $p$  and  $q$  are incompatible just in case *commitment to  $p$  precludes entitlement to  $q$* . Once we adopt this Brandomian frame, we can think about the necessary role that incompatibility plays in a discursive practice: it enables speakers to *challenge* the commitments of other speakers by making claims incompatible with them. I show through consideration of a dialogue that imagines the opposite, that, in order to play this challenge function, a symmetric structure of incompatibility is necessary. So, incompatibility, indeed must be symmetric. If we think of semantic contents as conferred by underlying normative practices, as Brandom does, this explains why an incompatibility relation between any two contents must be symmetric. I conclude by considering, in general terms, the methodology of drawing semantic conclusions from pragmatic premises in this way and how it affords us a new way of thinking about and constructing transcendental arguments.

## 2 The Incompatibility-Based Account of Negation

In a recent paper, Berto (2015), drawing on work from Dunn (1993, 1996) and Restall (1999), proposes an incompatibility-based account of negation,

now commonly known as the “Australian Plan” for negation (Meyer and Martin 1986, Berto and Restall 2019).<sup>1</sup> The core meaning of “not,” according to Berto, is to be understood in terms of the notion of incompatibility, which is treated as primitive for the purposes of laying down the semantics for negation. Berto writes,

I take (in)compatibility as the primitive twofold notion grounding the origins of our concept of negation and of our usage of the natural language expression ‘not’. Explanations stop when we reach concepts that cannot be defined in terms of other concepts, but only illustrated by way of example. A good choice of primitives resorts to notions we have a good intuitive grip of—and this is the case, I submit, with (in)compatibility.

It is difficult to think of a more pervasive and basic feature of experience, than that some things in the world *rule out* some other things; or that the obtaining of this precludes the obtaining of that; or that something’s being such-and-such excludes its being so-and-so, (768-769).

The concept of incompatibility appealed to here is, in the first instance, a *material* rather than *formal* notion.<sup>2</sup> It is a relation that obtains between concepts, judgments, properties, states of affairs, and so on, in virtue of their *material content* rather than their *logical form*. For instance, a monochromatic solid’s being red is incompatible with its being blue in the sense that its being red *rules out*, *precludes*, or *excludes* its being blue. This relation between its being red and its being blue is not, properly speaking, a *logical* relation between these two contents, like the relation between something’s being red and its not being red. On the contrary, this material incompatibility relation is appealed to in the semantics in order to specify the properly logical incompatibility relation that obtains between a sentence and its formal negation.

On the semantics proposed by Berto (2015), negation is taken to be a modal operator defined on the basis of this notion of incompatibility. Primitive compatibility relations are modeled by way of an accessibility relation between information-containing “states” (which one can hear as

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<sup>1</sup>It’s worth noting from the outset that, though the Australian Plan is the most prominent incompatibility-based account of negation and so the one on which I will focus here, it’s not the only such account. See also Francez (2019) and Brandom (2008, 2018). As far as I’m aware, all such accounts presuppose the symmetry of incompatibility, and so the argument developed here can be straightforwardly carried over to any such account.

<sup>2</sup>See Sellars (1953) for an influential discussion of this distinction.

shorthand for “states of affairs”) that are partially ordered by a relation of information inclusion  $\sqsubseteq$ .<sup>3</sup> That is,  $s \sqsubseteq t$  just in case  $t$  contains all of the information in  $s$ . We say that a sentence  $\varphi$  *holds* in a state  $s$ , by writing  $s \Vdash \varphi$ . We assume that, if  $p$  holds in  $s$ , and  $t$  includes  $s$ , then  $p$  holds in  $t$ . That is, we assume the following:

**Hereditary Condition:** If  $s \Vdash p$  and  $s \sqsubseteq t$ , then  $t \Vdash p$

We can now define compatibility as an accessibility relation  $C$  that satisfies the following constraint:<sup>4</sup>

**Persistence:** If  $sCt$  and  $s' \sqsubseteq s$  and  $t' \sqsubseteq t$ , then  $s'Ct'$

So, if  $s$  is compatible with  $t$ ,  $s$  contains all the information in  $s'$ , and  $t$  contains all the information in  $t'$ , then  $s'$  is compatible with  $t'$ . Essentially, this says that, if one state is compatible with another, then, if you remove information from these states, the resulting states continue to be compatible; one can only make compatible states incompatible by *adding* information to one, the other, or both of them.<sup>5</sup> Beyond this formal constraint, the notion of (in)compatibility is not defined, but, rather, appealed to as a primitive. The semantics for negation defined on the basis of this notion of incompatibility says that  $\neg\varphi$  holds in a state  $s$  just in case, for every state  $t$  such that  $s$  is compatible with  $t$ ,  $\varphi$  does not hold. That is:

**S<sub>¬</sub>:**  $s \Vdash \neg\varphi$  just in case  $\forall t(sCt \supset t \not\Vdash \varphi)$

To consider an example, “ $\neg(a$  is blue)” holds in a state consisting in  $a$ ’s being red, since every state with which this state is compatible—for instance, the state consisting in  $b$ ’s being blue, the state consisting in  $a$ ’s being red and  $c$ ’s being yellow, and so on—is a state in which “ $a$  is blue” fails to hold.

One important philosophical upshot of this account of negation—its main merit, as Berto lays it out—is that it makes possible a view that we might call “pros hen pluralism” about negation, according to which “not”

<sup>3</sup>These states are sometimes called “worlds,” but I avoid that terminology to avoid confusion with the possible worlds of classical semantics, the sort theorized about by Lewis (1986) and Stalnaker (1984), which are maximally determinate ways for the world to be. If one is to employ the terminology of “worlds” here, the “worlds” at issue are best understood as (at least potentially) *partial* worlds, non-maximal states of affairs that may be more or less determinate. Once again, to avoid confusion, I’ll just call them “states.”

<sup>4</sup>In using this terminology here, I follow Golan (2023), who draws it from Brandom (2008). Berto calls this constraint “Backwards.”

<sup>5</sup>It follows from this constraint that the null state, which contains no information, is compatible with every state that is compatible with some state.

is said in many ways by different logicians, but there is a core meaning of “not” that these different logicians all share, to which their different uses are all related.<sup>6</sup> On Berto’s account, the semantics just stated defines the *core* meaning of negation. *Secondary* meanings of negation, the different meanings of “negation” as the term is used by classical, constructive, and paraconsistent logicians, can all be understood in terms of the imposition of restrictions on the inclusion relation or accessibility relation which do not follow from the set-up itself. The classical logician imposes restrictions on the inclusion and accessibility relation that make it such that all states must be *complete* and *consistent*. The constructive logician allows states that are *incomplete*, allowing non-trivial inclusion relations between states, such that it is not the case that, for all states  $s$ ,  $s \Vdash p$  or  $s \Vdash \neg p$ . The paraconsistent logician allows states that are *inconsistent*, allowing states that are not compatible with themselves, and to which other states still stand in non-trivial inclusion relations, such that it is not the case that, for all states  $s$ , if  $s \Vdash p$  and  $s \Vdash \neg p$ , then  $s \Vdash q$ .<sup>7</sup> Though these logicians differ in taking negation to have different *secondary* meanings, insofar as all of these meanings are understood in terms of the imposition of restrictions on the models that are considered with respect to the same basic semantics, they all take negation to have the same *core* meaning.

Any laws of negation that directly follow from this core semantics will be taken to be the basic laws of negation, such that, if some “negation” operator does not satisfy them, it is not a *negation* operator. To consider such laws, we can define a notion of semantic entailment as follows:

**Semantic Entailment:**  $\varphi \vDash \psi$  just in case, for any state  $s$  such that  $s \Vdash \varphi$ ,  $s \Vdash \psi$ .

Simply in virtue of the core semantics for negation, the following entailment fact holds:

**Minimal Contraposition:** If  $\varphi \vDash \psi$ , then  $\neg\psi \vDash \neg\varphi$ <sup>8</sup>

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<sup>6</sup>The Greek phrase “pros hen” means “in relation to one.” The use of this term is, of course, a reference to Aristotle (See *Met.*  $\Gamma$  2), though one should not take the concept of a “pros hen” relation employed to be identical to that employed by Aristotle.

<sup>7</sup>Though, it should be noted, that some paraconsistent logicians might have trouble living happily in this semantic framework. For instance, the framework is not able to accommodate the negation of Preist’s (1979) LP, one of the more popular paraconsistent logics, since Minimal Contraposition, stated below, fails in LP. On this point, see De and Omori (2018: 287-288), and, for a suggested response, Omori and De (2022).

<sup>8</sup>The name “minimal contraposition” (Berto 2015) is perhaps a bit misleading here, since this contraposition principle is definitive of *subminimal* negation, with the definitive contraposition principle of minimal negation being the one that corresponds to the symmetry of incompatibility:  $\varphi \vDash \neg\psi \Leftrightarrow \psi \vDash \neg\varphi$  (See Dunn 1999: 32).

To see this, suppose  $\varphi \vDash \psi$ , so any state in which  $\varphi$  holds is a state in which  $\psi$  holds. Now suppose there is some state  $s$  such that  $s \Vdash \neg\psi$ , so there is no state compatible with  $s$  in which  $\psi$  holds. Since any state in which  $\varphi$  holds is a state in which  $\psi$  holds and there is no state compatible with  $s$  in which  $\psi$  holds, there is no state compatible with  $s$  in which  $\varphi$  holds. So  $s \Vdash \neg\varphi$ . Other than minimal contraposition, what else should be taken to be a basic law of negation, following from its core meaning? Berto suggests the following law:

**Double Negation Introduction:**  $\varphi \vDash \neg\neg\varphi$

However, unlike minimal contraposition, which follows directly from the formal set-up, something more must be imposed to get double negation introduction: the symmetry of incompatibility.

To treat incompatibility as a symmetric notion is to impose symmetry on the accessibility relation  $C$ , such that, if  $t$  stands in  $C$  to  $s$ , then  $s$  stands in  $C$  to  $t$ . If we do *not* impose symmetry on the accessibility relation, a different notion of negation can be defined, for which Berto uses the symbol “ $\sim$ ” rather than “ $\neg$ ”:

**S<sub>~</sub>:**  $s \Vdash \sim\varphi$  just in case  $\forall t(tCs \rightarrow t \not\vDash \varphi)$

So, to take the same example, “ $\sim(a \text{ is blue})$ ” holds in a state consisting in  $a$ ’s being red, since every state that is compatible with this state—for instance, the state consisting in  $b$ ’s being blue, the state consisting in  $a$ ’s being red and  $c$ ’s being yellow, and so on—is a state in which “ $a$  is blue” fails to hold. It’s hard to resist the conclusion that “ $\neg(a \text{ is blue})$ ” and “ $\sim(a \text{ is blue})$ ” mean the very same thing: the thing would be expressed in English as “It’s not the case that  $a$  is blue.” If compatibility is symmetric, of course, they do mean the same thing; they hold in just the same states.

If we impose symmetry, such that “ $\neg$ ” and “ $\sim$ ” express the same operation (namely, negation), that double negation introduction must hold. To see this, note first that  $\neg$  and  $\sim$  are related in the following way:

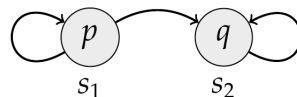
**Galois Connection:**  $\varphi \vDash \neg\psi$  just in case  $\psi \vDash \sim\varphi$

Now, from the trivial entailments,  $\neg\varphi \vDash \neg\varphi$  and  $\sim\varphi \vDash \sim\varphi$ , we have the following entailments:

**DNI<sub>A</sub>:**  $\varphi \vDash \sim\neg\varphi$

**DNI<sub>B</sub>:**  $\varphi \vDash \neg\sim\varphi$

So, since, if we have symmetry, such that “ $\neg$ ” and “ $\sim$ ” express the same operation, we have double negation introduction. Alternately, if we *don't* have symmetry, we *don't* have double negation introduction. To see this, consider the following set of states, with the arrows showing relations of compatibility between them:



Both  $s_1$  and  $s_2$  are compatible with themselves,  $s_1$  is compatible with  $s_2$ , but  $s_2$  is not compatible with  $s_1$ . To see that  $p \vDash \neg\neg p$  is invalidated by this model, first, see that  $p$  holds in  $s_1$ . Now, see that  $\neg p$  holds in  $s_2$ , since  $s_2$  is compatible with no states in which  $p$  holds. Finally, see that  $\neg\neg p$  does not hold in  $s_1$ , since  $\neg\neg p$  holds in  $s_1$  just in case  $s_1$  is compatible with no state in which  $\neg p$  holds, but  $s_1$  is compatible with a state in which  $\neg p$  holds, namely  $s_2$ . Thus, in this model:  $p \not\vDash \neg\neg p$ . If incompatibility must be symmetric, and the core meaning of negation is defined on the basis of incompatibility in this way, then any negation operator, as such, must satisfy double negation introduction.

### 3 The Question of Symmetry

But *must* (in)compatibility be symmetric? Could there be contents that stand in asymmetric (in)compatibility relations? Berto (2015) claims that there cannot be:

Now (in)compatibility must be symmetric: whatever ontological kinds  $a$  and  $b$  belong to, it appears that if  $a$  rules out  $b$ , then  $b$  has to rule out  $a$ ; that if  $a$ 's obtaining is incompatible with  $b$ 's obtaining, then  $b$ 's obtaining must also be incompatible with  $a$ 's obtaining; etc (779).

Now, presumably Berto is not really attempting to give an argument here; he's simply appealing to our supposed intuition that incompatibility must be symmetric. Still, given what he says, he might be construed as arguing inductively along the following lines. Pick some  $a$  and some  $b$ , be they properties, states of affairs, propositions, event-types, or what have you, such that  $a$  rules out  $b$ . Now confirm that  $b$  also rules out  $a$ . Repeat, picking

$a$  and  $b$  from different ontological categories until you are satisfied that (in)compatibility, in general, must be symmetric.

It turns out, however, that not everyone has found it to be clear that, for any  $a$  and  $b$  you pick, if  $a$  rules out  $b$ , then  $b$  rules out  $a$ ; counter-examples have been proposed. For instance, if we think of *prevention* as a kind of incompatibility, such that  $a$ 's preventing  $b$  is a way for  $a$  to be incompatible with  $b$ , then we get counter-examples.<sup>9</sup> Dunn (1996) gives the following example: Jon's practicing his saxophone prevents his father from reading a technical paper, but Jon's father's reading a technical paper does not prevent Jon from practicing his saxophone, (13-14). Now, Berto (2015) and then again Berto and Restall (2019) in response to De and Omori (2018) correctly, I think, diagnose the example as hinging on a conflation of the asymmetric causal relation of prevention and the symmetric non-causal relation of incompatibility. We must be able to tease these apart because a fact consisting in the former relation obtaining between two event-types might be explained by a fact consisting the latter relation obtaining between those two event-types. That is, an event-type  $a$  might be incompatible with an event-type  $b$  *because* the occurrence of  $a$  prevents the occurrence of  $b$ . If the fact that the occurrence of  $a$  prevents the occurrence of  $b$  *explains* the fact that  $a$  is incompatible with  $b$ , then the former fact cannot be *identical to* the latter fact.<sup>10</sup> And it seems that we have this explanatory asymmetry in the proposed counter-example. Jon's practicing the saxophone is incompatible with his father's reading the paper *because* the occurrence of the first event-type prevents the occurrence of the second event-type. It follows from this that the prevention relation here is not identical to the incompatibility relation, and this allows us to say that it is only the prevention relation here (which explains the obtaining of the incompatibility relation) that is asymmetric; the incompatibility relation that obtains between these two event-types (which is explained by the prevention relation's obtaining between them) is symmetric.

Still, even supposing that this response to the proposed counter-example is successful, nothing about it is sufficiently general to ensure that other counterexamples won't arise. In response to the saxophone example above, Berto says "Considerations involving asymmetrical causal relations should not sneak into the purity of our intuitions on the symmetry of (in)compatibility," (780). But it's not hard to construct cases that do not

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<sup>9</sup>See Hartonas and Dunn (1993: 14-15).

<sup>10</sup>This is an instance of the Euthyphro schema: If someone's being pious explains their being loved by the gods, then their being pious cannot be identical to their being loved by the gods. Schematically, if some fact  $A$  explains some fact  $B$ , then  $A$  cannot be identical to  $B$ .



hinge on asymmetric causal relations which cast doubt on how pure our intuitions really are. For instance, most people that I've asked seem to think that one person *a* can be romantically compatible with another person *b*, yet *b* can be romantically incompatible with *a*.<sup>11</sup> Here, the ontological kinds are people, and the (in)compatibility relations are interpersonal ones between people. What should we say here? It's not at all intuitively clear. One response to this example that preserves the intuition that there really can be asymmetric romantic incompatibility relations involves further specifying what the relevant notion of incompatibility for the Australian Plan is such that either people are not the right sort of relata or that romantic (in)compatibility is not the right sort of relation. In particular, one might say that the relevant concept of (in)compatibility is the concept of a relation that obtains between *contents*, and people (unlike properties, propositions, or states of affairs) neither are nor have contents, so though there may be some sort of (in)compatibility relation that obtains between people that is somehow *analogous* to the relevant (in)compatibility relation that obtains between contents, it is not the very same relation. An alternate response to this example—not necessarily incompatible with the main sentiment of the first—is to say that, in such a case of apparently asymmetric romantic (in)compatibility, there are really *two* (in)compatibility relations—one for each person, given what matters to them in a relationship—and each one is symmetric.<sup>12</sup> Thus, *a* is (symmetrically) romantically compatible with *b*, given the (in)compatibility relation indexed to him, and *b* is (symmetrically) romantically incompatible with *a*, given the (in)compatibility relation indexed to her. Perhaps this is the way to go. Still, whatever we say, it seems to require some substantive theorizing; we cannot simply rely on the “purity of our intuitions.”

To get to the real point here, even if we *do* have something to say about all of the cases that have been brought up that allows us to maintain that incompatibility is, in each genuine case, symmetric, it seems that *still*, all we can really conclude here is that it *appears* that incompatibility must be symmetric; we are not entitled to conclude that incompatibility, as such, *really must* be symmetric.<sup>13</sup> Entitlement to that latter claim requires one to

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<sup>11</sup>Thanks to Arianna Lombardi for suggesting this example.

<sup>12</sup>Thanks to an anonymous referee for suggesting this response, drawing on Lewis's (1968, 1986) reflections on the asymmetry of counterpart relations but symmetry of similarity relations.

<sup>13</sup>There is a whole class of potential counter-examples that I am not considering here of concern in the natural language semantics for conditionals and epistemic modals, in which the incompatibility relation defined by semantic theories actually is asymmetric, and that is in dynamic semantic theories meant to accommodate data like Reverse Sobel

be able to say *why* incompatibility must be symmetric, and neither Berto nor anyone else, as far as I'm aware, says anything to answer this question. Now, Restall's (1999) language is much less committal with respect to the question of whether there could be asymmetric incompatibility relations. Laying out the same basic semantics, Restall says,

We can consider some properties which it would be plausible to assume that  $C$  has. For example, compatibility certainly does *seem* to be symmetric. That is, if  $xCy$  then  $yCx$ , (62).

Here, Restall is more or less explicit that he is laying down symmetry as a constraint on the accessibility relation as a plausible assumption for doing formal semantics. Insofar as this is what one is doing, there is no problem here. However, there *is* a problem for the incompatibility-based account of negation insofar as it is proposed as a genuine account from which the basic features of negation follow. Berto purports to be giving an account according to which "nothing can be called a negation properly if it does not satisfy (Minimal) Contraposition and Double Negation Introduction," (761). As we've seen, double negation introduction holds just in case symmetry is imposed as a constraint on the accessibility relation  $C$ . So, if one purports to be giving an account of negation in terms of incompatibility, and one holds that it is an essential feature of something's genuinely counting as a negation operator that double negation introduction holds, then one had better be able to say why incompatibility must be symmetric. But neither Berto, Restall, nor anyone else who has proposed this incompatibility-based account of negation, has anything to say here.

It is worth being explicit that, whatever the problems with the classical conception are, the classical logician clearly does not have this problem.<sup>14</sup> It is clear, on the classical conception, what the laws of negation are, double negation introduction being one of them.  $p$  entails  $\neg(\neg p)$  because, if  $p$  is

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Sequences for conditionals or data involving epistemic "might." For conditionals, see, for instance, von Fintel (2001), Gillies (2007), Willer (2017) and, for epistemic modals see especially Lennertz (2018). I don't consider these kinds of cases here because the semantic *values* defined in these sorts of dynamic theories of that support asymmetric incompatibility relations can't be identified with semantic *contents*, as one does in a standard truth-conditional semantics. This gets back to the point mentioned above, that we need to be clear about just what the relata of the relevant incompatibility relation actually are.

<sup>14</sup>Nor, we should note, does the non-classical logician who follows the "American plan," thinking of negation as a contradictory forming operator as the classical logician does, but distinguishing untruth from falsity so as to maintain that a sentence might be neither true nor false or both true and false. See De and Omori (2018, 292-296) on this point.

true, then  $\neg p$  is false, and if  $\neg p$  is false, then  $\neg(\neg p)$  is true. This fact holds in virtue of the account of the basic semantic function of the negation operator. On the classical conception, what the negation operator does, when prefixed to a sentence that is either true or false, is form a sentence that has the opposite truth value, *false* if the original sentence is true, *true* if the original sentence is false. Double negation introduction follows directly from this conception of what a negation operator is and does. Of course, lots of other laws also follow it, for instance, the much more contentious double negation elimination, a “law” that our constructive logician rejects. However, if one wants to employ a weaker logic, one in which some of these classical “laws” of negation are not laws, one has to be able to give an account of the negation operator that one is using such that it is clear, according to that account, what the laws of negation are. Berto claims to be able to do this. However, insofar as he has nothing to say as to why it is that incompatibility must be symmetric, he is not entitled to this claim.

Now, at this point, it might seem that Berto can retreat and revoke his commitment to the claim that he has really offered an account of negation according to which nothing can be called a negation properly if it does not satisfy double negation introduction. So-called “sub-minimal” negations have been explored which do not validate double negation introduction (Dunn 1993; Hazen 1995). It seems to me at least, however, that a minimal negation operator really is the *minimal* negation operator, in the sense that nothing weaker than it really is a negation operator. Here’s a simple argument for this claim: If  $p$ , then clearly not *not*  $p$ . After all,  $p$ . So, not *not*  $p$ . If one gives an account of the propositional operator expressed by “ $\neg$ ,” such that this argument is no good when one substitutes one’s “ $\neg$ ” for “not,” then, whatever one’s “ $\neg$ ” means, it doesn’t mean *not*. Now, a defender of asymmetric incompatibility relations might take themselves to be able to reconstruct this reasoning by insisting that the English word “not” is equivocal between two negations,  $\neg$  and  $\sim$ , since, after all,  $p$  entails  $\sim\neg p$ . Such a line, however, is clearly not open to Berto. Insofar as Berto purports to be giving an account of *negation*, he’s clearly committed to the claim that there is *one thing* that properly bears that title, namely, *negation*. If we want to vindicate the incompatibility-based account of negation, we must be able to say why incompatibility must be symmetric.

## 4 Two Attempts at an Answer

Before going on to propose the answer to the question of why incompatibility must be symmetric I ultimately will endorse, I want to consider two

alternative attempts to answer the question that I think fall short, but in illuminating ways. The first is a deflationary response according to which this question is simply the result of a particular way of formally representing incompatibility.<sup>15</sup> To introduce this response by way of analogy, consider first the relation of siblinghood. In first-order logic, we might represent siblinghood as a binary relation  $S$  that two individuals  $x$  and  $y$  might stand in, and we might then observe that this relation is symmetric: for all  $x$  and  $y$ , if  $Sxy$  then  $Syx$ . If one asks for an explanation of *why* siblinghood is symmetric, the proper response is to say that the question only arises in the context of a particular inperspicuous formal representation of siblinghood. Really, two, three, or any number of people are siblings just in case they have the same parents.<sup>16</sup> The property of having the same parents, in terms of which the relation of siblinghood is defined, is a property that applies to *pluralities* of people. For any number of people who all have the same parents, they are all siblings. The relation of siblinghood is thus symmetric simply because it is defined in terms of a property of pluralities, and inclusion in a plurality is indifferent to order. By analogy, we might say that the relation of incompatibility is symmetric because it is defined in terms of a conceptually prior property that applies to *pluralities* of states. It is convenient, in certain formal frameworks, to represent incompatibility as a binary relation between states. However, just as one should not read too much into the metaphysics of siblinghood by a representation of it in first-order logic as a binary relation, one should not read too much into the metaphysics of incompatibility by a representation of it in a certain formal framework as a binary relation between states.

Now, the deflationary approach only gets its plausibility insofar as, like the “relation” of siblinghood which we can define in terms of the property of having the same parents, we can specify a property of pluralities in terms of which we can define the relational notion of incompatibility that figures in the semantics. Intuitively, there’s a clear candidate: a plurality of states are incompatible just in case it is impossible for all of them to obtain together.<sup>17</sup> That is, the incompatibility of a plurality of states is their *joint impossibility*. To define this notion of joint impossibility precisely, we might follow Fine (2017) and define the *fusion* of a plurality of states  $s_1 \sqcup s_2 \dots \sqcup s_n$  as the minimally informative state that contains the information of all of those states.<sup>18</sup> We can then say that a plurality of states

<sup>15</sup>I would like to thank an anonymous referee for raising this possible response.

<sup>16</sup>I am ignoring the case of half-siblings for the purpose of this example.

<sup>17</sup>This way of defining incompatibility between states is suggested, among many other places, in Berto and Jago (2018, 76).

<sup>18</sup>Officially  $u = s \sqcup t$  just in case  $s \sqsubseteq u$ ,  $t \sqsubseteq u$ , and for all  $u'$  such that  $s \sqsubseteq u'$  and  $t \sqsubseteq u'$ ,

are jointly impossible just in case their fusion is impossible, and in just such a case, we say that they are incompatible. If one defines incompatibility in this way, then the question of the symmetry of incompatibility is immediately answered. If  $s$  is incompatible with  $t$  just in case their fusion is impossible, then, since there is just one state that is their fusion,  $s$  is incompatible with  $t$  just in case  $t$  is incompatible with  $s$ . In addition to answering our question, this approach also has the virtue of generalizing incompatibility beyond simply a binary relation between states, accommodating, for instance, “inconsistent triads” of states such as  $a$ ’s being red,  $a$ ’s being ripe, and  $a$ ’s being a blackberry, recently brought to attention in this context by Golan (2023).<sup>19</sup> Any two of these states are compatible with each other, but the three are “ternarily incompatible” in that any two of them together is incompatible with the third. This is understood, on this approach, in terms of the fact that the fusion of all three is impossible, and, once again, symmetry of incompatibility trivially follows.

Though this deflationary response provides a simple answer to our question, there are substantial reasons to think that, unlike siblinghood, the conceptually primary notion of incompatibility really is that of *relation between* states rather than a *property of pluralities* of states. Let me give just one argument which I think suffices for the basic conceptual point. Suppose  $s$  and  $t$  are incompatible. Suppose, for instance, that  $s$  is the state of  $a$ ’s of being red all over and  $t$  is the state of  $a$ ’s being blue all over. The incompatibility between these states is understood, on this approach, in terms of the impossibility of their fusion  $s \sqcup t$ ,  $a$ ’s being red all over and blue all over. So far so good. But now consider the fusion of this state with an unrelated state  $u$ —say,  $a$ ’s being a square. The state  $s \sqcup t \sqcup u$  is also impossible, and so it will follow, on this approach, that something’s being red all over and blue all over is incompatible with it’s being a square. But this doesn’t seem right. Though something’s being red all over and blue all over is a state that is *self*-incompatible, it’s both part of our intuitive understanding of incompatibility and an important feature of the formal framework under consideration here that a state’s being self-incompatible

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$u \sqsubseteq u'$ . I appeal to this construction from Fine just to make the proposal determinate; I don’t intend to commit myself to any features of Fine’s semantic framework here, and the argument that follows doesn’t hinge on the formal detail of defining fusion in this way. For a framework that combines core ideas of Fine’s truthmaker semantics and the Australian Plan under discussion here, see Plebani, Rosella, and Saitta (2022).

<sup>19</sup>Though the Australian Plan officially includes only a binary relation, Golan (2023) argues that a proponent of the Australian Plan semantics for negation ought to extend their notion of (in)compatibility beyond a merely binary relation and include a ternary relation as well, as doing so enables a unified incompatibility-based semantics for relevant negation and relevant conditional.

doesn't mean that it is incompatible with every other state.<sup>20</sup> Still, though it's wrong to say that  $a$ 's being blue all over and red all over is incompatible with its being a square, it's clearly right to say that  $a$ 's being blue all over and a square is incompatible with its being red all over (after all, in the first state, it's blue all over, and that rules out its being red all over).<sup>21</sup> So, schematically, though  $s$  together with  $t$  is *not* incompatible with  $u$ ,  $s$  together with  $u$  is incompatible with  $t$ . And this just means that the incompatibility of  $s \sqcup u$  and  $t$  can't be understood in terms of the impossibility of their fusion, since the fusion of these states is the very same state as the fusion of  $s \sqcup t$  and  $u$ , two states which aren't incompatible. So, we can't, in general, understand the incompatibility relation between states in terms of the impossibility of their obtaining together.

Now, perhaps there's a different version of the deflationary response that's not subject to this line of criticism. However, examples like this suggest that the basic concept of incompatibility appealed to in the incompatibility-based account of negation really is the concept of a relation between states rather than the concept of a property of collections of states. And, of course, this is just how Berto articulates it. Recall his intuitive explication of incompatibility in terms of the fact that "some things in the world *rule out* some other things; or that the obtaining of this *precludes* the obtaining of that; or that something's being such-and-such *excludes* its being so-and-so," (768-769). The vocabulary that Berto deploys and emphasizes here—that of "ruling out," "precluding," and "excluding"—is *essentially relational*. I take it that this reflects the correct metaphysics of incompatibility. As such, the question about the symmetry of this relation does seem like a genuine question.

I turn now to a second attempt at an answer. Note first that the example above involved the consideration self-incompatible, or, as I'll say *incoherent* states. The strategy of this second answer is to argue, first, that, at least for *coherent* states, incompatibility must be symmetric, and then argue that incompatibility relations between incoherent states depend on incompatibility relations between the coherent states that are informationally contained within them. This may seem to be a promising strategy

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<sup>20</sup>If a state's being self-incompatible implied that it was incompatible with every other state, this would mean that the negation of every sentence holds at any self-incompatible state, and this is a kind of explosion that we want to avoid insofar as we're interested in accommodating any sort of paraconsistent reasoning.

<sup>21</sup>One way to pump the intuition of a difference here is to consider Sylvan's Box-type cases (Priest 1997). Case one: you open a box and there's a red and blue object. Is it a square? Well, maybe. Case two: you open a box and there's a blue square. Is it red? No, it's blue.

since it seems that there's a straightforward argument for the first point that, for coherent states, incompatibility must be symmetric. Suppose for contradiction that  $s$  and  $t$  are coherent,  $s$  is incompatible with  $t$ , and  $t$  is compatible with  $s$ . If  $t$  and  $s$  are coherent, and the obtaining of  $t$  doesn't exclude the obtaining of  $s$ , then it seems clear that the fused state of affairs consisting in  $t$  along with  $s$  must also be coherent.<sup>22</sup> But if  $s$  is incompatible with  $t$ , that is, if  $s$  excludes  $t$ , then it seems that the fusion of  $s$  and  $t$  must be incoherent. After all, given the principle of Persistence stated above, if  $s$  is incompatible with  $t$ , then, since  $s \sqcup t$  is a state that contains more information than  $s$ , then  $s \sqcup t$  is incompatible with  $t$ , and, by the same reasoning with respect to  $t$ ,  $s \sqcup t$  is incompatible with  $s \sqcup t$ .<sup>23</sup> So, given our supposition, the fusion of  $s$  and  $t$  both is and isn't coherent. Contradiction. So, if  $s$  and  $t$  are coherent, any (in)compatibility relation between them must be symmetric. One can then go on to argue any incompatibility between incoherent states is always a consequence of incompatibilities between coherent states they contain, and, as such, must be symmetric as well.

In evaluating this argument, the principle I want to focus on is that if  $s$  is incompatible with  $t$ , then  $s$  and  $t$  together are still incompatible with  $t$ . This principle might seem unassailable, but what, exactly, is the reasoning for it? As it occurs in the above argument with reference to Persistence, its justification is understood in terms of its being an instance of the more general principle of the *monotonicity* of incompatibility, that if  $s$  is incompatible with  $t$ , then  $s$  together with  $u$  is incompatible with  $t$ . There is reason to think, however, that this more general principle is simply an imposition of the formal framework that we've laid out, and it cannot be appealed to in order to make a conceptual point about the notion of incompatibility on which the framework is based. The point that we must disentangle features of the rendering of incompatibility in the formal framework from features of the concept of incompatibility itself will be familiar from our consideration of the deflationary approach just considered. It is thus important to point out that incompatibility is *not*, in general, monotonic. In making this point, it's important to be clear once again that the primary notion of incompatibility that we're dealing with here is *material* incompatibility, not formal incompatibility, and there are clear examples of material in-

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<sup>22</sup>Note, we are not saying as we did above that  $t$ 's being compatible with  $s$  just is  $t \sqcup s$ 's being coherent, but, if  $t$  and  $s$  are themselves coherent, the coherence of their fusion at least seems to be a necessary condition of their compatibility.

<sup>23</sup>Persistence says that if  $s$  and  $t$  are compatible, you can't make them incompatible by subtracting information from them. It follows directly that you can't make incompatible states compatible by adding information from them.

compatibility relations in which monotonicity fails.<sup>24</sup> For instance, Sadie's being a mammal is incompatible with her laying eggs, but her being a mammal and being a platypus isn't incompatible with her laying eggs. Indeed, it seems that for *most* examples of material incompatibility that we're naturally inclined to think up, we can find defeaters. For instance, to take the example considered above, Jon's practicing the saxophone is incompatible with his father's reading a technical paper, but his practicing the saxophone and doing so with a mute in it is not incompatible with his father's reading a technical paper. It's worth noting that every participant to the debate has been happy to countenance this example as a genuine case of incompatibility of the relevant sort. Of course, one might retract this acceptance, but, if you rule out all such examples, you're going to end up saying no claims, except, perhaps claims about the colors and shapes of monochromatic solids, are really incompatible.

Now, once we drop this assumption of the monotonicity of incompatibility, we are no longer operating within the formal framework of the Australian Plan. One move a proponent of the Australian Plan such as Berto might make at this point, then, is to say that insofar as we *are* operating within the formal framework of the Australian Plan, the argument above suffices, since the incompatibility relation of the formal framework is monotonic. That is fair enough as far as it goes. Still, if one stops there, the deeper conceptual question about the basic concept of material incompatibility on which the framework is based will remain unanswered. Thus, if we consider different formal frameworks for defining negation in terms of incompatibility which *permit* non-monotonic incompatibility relations, we will face the same question.<sup>25</sup> As we've just suggested, there are good reasons to actually consider such frameworks. Indeed, once one recognizes that the vast majority of material incompatibility relations are non-monotonic, it is natural to wonder whether the Australian Plan semantics for negation, which imposes the monotonicity of incompatibility in the very set-up, is really the best formal framework for thinking about the meaning of negation in terms of material incompatibility. I will bracket this question of choice of formal framework here, as, once again, my concern is simply with the basic conceptual question of whether the notion of incompatibility—which may be appealed to in a number of different for-

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<sup>24</sup>For discussion of this point, see Nickel (2013: 346-357), Hlobil (2016), and Brandom (2018).

<sup>25</sup>For formal semantic frameworks in which negation is understood in terms of potentially non-monotonic relations of incompatibility, see, for instance, the phase space semantics proposed by Porello (2012) or Kaplan (2021), or the truth-maker semantics proposed by Hlobil (2023).



mal frameworks—really must be symmetric. In order to finally answer this question, I suggest we think about incompatibility somewhat differently than Berto proposes we do.

## 5 The Normative Pragmatic Conception

In explicating the concept of material incompatibility or exclusion, Berto (2008) makes it clear that the notion is to be understood *semantically*, in terms of *contents*, rather than *pragmatically*, in terms of *acts*. He tells us:

Put it any way you like, material exclusion has to do with *content*, not mere performance: it is rooted in our experience of the world, rather than in pragmatics, (Berto 2008, 180).

I have followed this lead above, thinking of the relation of the incompatibility relation as states of affairs, presumably, consisting in objects having properties and standing in relations. From this perspective, the question of why incompatibility must be symmetric seems to me to be pretty intractable. It seems to require saying something in general about what it is for something to be a state of affairs, such that, saying this, we are able to say that, for any two possible states of affairs  $s$  and  $t$ , if  $s$  is incompatible with  $t$ , then  $t$  is incompatible with  $s$ . If we continue to think in these terms, I do not know what further lines of thought about the nature of states of affairs could entitle us to this conclusion. In order to make progress on the question of the symmetry, I think that this assumption that Berto makes here is precisely the one that we must reject.

Against Berto, I suggest that we don't think about incompatibility, in the first instance, as an *alethic modal* relation between *contents*, such as states of affairs, but, rather, as a *normative* relation between *acts*, specifically, acts of *making claims*.<sup>26</sup> Now, of course, acts of making claims have contents—in making a claim one expresses a proposition—and such a proposition will be true just in case a certain state of affairs obtains. However, the thought I want to pursue here, owed to Brandom (and his Hegel) (2008, 2019), is that we can only understand the *alethic* incompatibility relations that obtain between these states of affairs by thinking through the *normative* incompatibility relations that obtain between the claims that are made true

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<sup>26</sup>It's worth noting that, in other work regarding the "bilateral" reading of multiple conclusion sequent calculi, Restall (2005) adopts a normative conception of incompatibility quite like the one I am about to suggest. See Hlobil (2023) for a complementary proposal relating Restall's normative bilateralism to truth-maker semantics, articulated in alethic modal terms.

by the obtaining of these states of affairs. I am proposing, then, that we think about incompatibility, in the first instance, not in terms of what it is for two *worldly entities* to be *alethically* incompatible, but in terms of what it is for two *discursive acts* to be *normatively* incompatible. I will return to the question of why this does not constitute a mere changing of the topic in the final section of this paper. First, however, I want to show what progress can be made on our question once we switch conceptual frames.

To get into the frame that I'm suggesting, note that just as there is a perfectly good sense something's *being* red rules out, precludes, or excludes its being blue, there's a perfectly good sense in which *claiming* that something's red rules out, precludes or excludes claiming that it's blue. It's, of course, not *impossible* to claim that something's blue while continuing to claim that it's red. Rather, it's *impermissible*. To put this impermissibility in the normative pragmatic terms laid out by Brandom (1994), being *committed* to the claim "*a* is red" precludes one from being *entitled* to the claim "*a* is blue." More generally:

In practical terms of normative status, to treat *p* and *q* as incompatible claims is to take it that commitment to one precludes entitlement to the other, (1994: 115).

As Brandom defines it here, incompatibility is symmetric: two claims are incompatible just in case commitment to *one* precludes entitlement to *the other*. But here too, this way of thinking about incompatibility does not itself mandate symmetry; the logically prior concept with which Brandom is working is what I'll call "preclusive consequence," where *p* stands in a relation of preclusive consequence to *q* just in case commitment to *p* precludes entitlement to *q*. Incompatibility is defined as preclusive consequence in both directions, but the concept of preclusive consequence in terms of which incompatibility is defined does not, as such, mandate symmetry. The question, translated into this idiom, is: must preclusive consequence be symmetric? Is it possible for there to be two claims, *p* and *q*, such that commitment to *p* precludes entitlement to *q* but not vice versa? Brandom, like Berto, says nothing to answer this question, simply assuming without argument that preclusive consequence is symmetric. However, this question, I now hope to show, is much more tractable.

The first step to answering this question is to answer why there must be preclusive consequence relations in the game of giving and asking for reasons at all. What is the role that these consequence relations play in the game of giving and asking for reasons? Before we consider whether the game could be played with these relations being asymmetric, let us

consider whether the game could be played without them at all. I submit that it could not be. Without preclusive consequence relations, making a move could not be counted as *challenging* another move. To make a move is to undertake a *commitment* to demonstrate one's *entitlement* to that move. Without challenges in the practice, which compel one to demonstrate one's entitlement to a move, the very idea that what one is doing in making a move is undertaking a commitment is lost. Challenges are, in a sense, the keystone that holds together the structure of the game of giving and asking for reasons. If there is no giving of reasons, there is no reasoning, and there is no giving of reasons if there is no calling for them.

What is the most basic case of a challenge? A challenger, in a way that is directed at the challengee, makes a claim that is incompatible with the one that they intend to challenge.<sup>27</sup> Where incompatibility is understood as an invertible relation of preclusive consequence, this makes sense. Consider two discursive practioners, Maddy and Norm. If Maddy makes the claim  $q$ , and Norm makes the claim  $p$ , where it is mutually acknowledged that commitment to  $p$  precludes entitlement to  $q$  and commitment to  $q$  precludes entitlement to  $p$ , it is mutually acknowledged by Maddy and Norm that the commitments they have respectively undertaken are normatively impossible: they cannot both be taken on. Insofar as making a claim paradigmatically does not only aim to *entitle* other players to it but also puts a demand upon them to *commit* themselves to it, Maddy's making  $q$  and Norm's making  $p$  leads to a situation in which it is mutually acknowledged that Maddy must show that her claim to being entitled to  $q$  is stronger than Norm's claim to being entitled to  $p$ , or else Maddy must revoke her commitment to  $q$ , and likewise for Norm. But what happens if Maddy takes commitment to  $p$  to preclude entitlement to  $q$  but not vice versa? Well, let's see.

## 6 A Dialogue

MADDY:  $q$

NORM: So, you are committed to  $q$ ?

MADDY: Yup.

NORM: Well, I challenge this commitment of yours on the following grounds:  $p$ ,  $p$  is incompatible with  $q$ , and my grounds for  $p$  are better than your grounds for  $q$ .

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<sup>27</sup>See Wanderer (2010) on the way in which a challenge must be second-personally directed.

MADDY: I agree that  $p$  is incompatible with  $q$ , and you're grounds for  $p$  are better than my grounds for  $q$ .

NORM: Surely, then, you must revoke your commitment to  $q$ .

MADDY: No. I will, however, commit myself to  $p$  now, since you've given good grounds for  $p$ .

NORM: How can you take yourself to be able to do that? You just agreed with me that  $p$  is incompatible with  $q$ , so, insofar as you are committed to  $q$  you are precluded from being entitled to  $p$ . So you *can't* commit yourself to  $p$ .

MADDY: That is a complete non-sequitur.  $p$  is incompatible with  $q$ , so, commitment to  $p$  precludes entitlement to  $q$ , but  $q$  isn't incompatible with  $p$ , so commitment to  $q$  does not preclude entitlement to  $p$ . Accordingly, I can commit myself to  $p$ , and that's what I just did.

Maddy takes  $p$  to be asymmetrically incompatible with  $q$ . So, though she takes commitment to  $p$  to preclude entitlement to  $q$ , she doesn't take commitment to  $q$  to preclude entitlement to  $p$ . Accordingly, though she takes it that someone who is committed to  $p$  is precluded from being able to commit themselves to  $q$ , she takes herself to be able to take on a commitment to  $p$ , even given a commitment to  $q$ . And that's just what she does. Is this coherent? Norm is keen to insist that it isn't:

NORM: No! You *can't* do that.

MADDY: Why not?

NORM: You grant that commitment to  $p$  precludes entitlement to  $q$ , right?

MADDY: Right.

*B* And you're committed to both  $p$  and  $q$ , right?

MADDY: Right.

NORM: So, you're both committed and precluded from being entitled to  $q$ ! You have an incoherent set of commitments, and you have an incoherent set of commitments in virtue of commitment to  $p$  and  $q$ . Now, you've already acknowledged that the grounds for  $p$  are much better than the grounds for  $q$ . Accordingly, since you must rectify your incoherent set of commitments, you must revoke your commitment to  $q$ .

Norm appears to point out a fatal incoherence in Maddy's position. Maddy takes her commitment to  $q$  to leave open entitlement to  $p$ , and so takes herself to be able to commit herself to  $p$ , even given a commitment to  $q$ . However, Maddy agrees that commitment to  $p$  precludes entitlement to  $q$ . Accordingly, it seems that Maddy's taking on a commitment to  $p$  in addition to  $q$  will result in her being *committed* to something to which she is *precluded from being entitled*, namely  $q$ . It might seem that the dialogue should end here. However, as foreshadowed by our discussion of non-monotonicity above, Maddy has a surprising response:

MADDY: Once again, that is a complete non-sequitur. Although commitment to  $p$  alone precludes entitlement to  $q$ , commitment to  $q$  and  $p$  together does not preclude entitlement to  $q$ . So, since I was committed to  $q$  already, and commitment to  $q$  does not preclude entitlement to  $p$ , I could commit myself to  $p$ , without precluding myself from being entitled to  $q$ , and that's just what I've done. Once I did this, of course, I was committed to  $p$ , but, because commitment to  $q$  and  $p$  together does not preclude entitlement to  $q$ , I am not precluded from being entitled to  $q$ .

NORM: Let me get this straight: You can be committed to  $q$  while also being committed to  $p$  because you're committed to  $p$  and  $q$ , and commitment to  $p$  and  $q$  together does not preclude entitlement to  $q$ , but I can't be committed to  $q$ , because I'm precluded from being entitled to it, since I'm committed to  $p$  without being committed to  $q$ , and commitment to  $p$  precludes entitlement to  $q$ .

MADDY: Exactly.

Insofar as Maddy takes commitment to  $p$  to preclude entitlement to  $q$  and also takes it that, given a commitment to  $q$ , she can commit herself to  $p$ , she must also take it that, though commitment to  $p$  alone precludes entitlement to  $q$ , commitment to  $p$  and  $q$  together does not. This is clearly a very strange attitude to have. However, strange as this attitude may seem, it's not immediately clear just what about it makes it incoherent.

To see what is ultimately wrong with Maddy's attitude, we must see what Norm is able to do, given that Maddy has this attitude. Let us continue the dialogue:

NORM: Ok, well then I revoke my commitment to  $p$ .

MADDY: Why would you do that? You have good reasons for  $p$ .

NORM: Sure, I have good reasons, but certainly it must be permissible, in response to a challenge to revoke one's commitment to the challenged claim. A challenge essentially presents me with a *choice*: to either give reasons for my challenged claim or to revoke my commitment to it. I opt for the latter.

MADDY: Alright, sure, that's indeed something you can do.

NORM: Ok, so I revoke my commitment to  $p$ . I now accept  $q$ .

MADDY: You accept  $q$  rather than  $p$ ? But your reasons for  $p$  are *better* than your reasons for  $q$ .

NORM: Well, that's no problem now that I've revoked my commitment to  $p$  and accepted  $q$ . Reasons *for*  $p$  are only reasons *against*  $q$  insofar as I'm committed to  $p$ . Now that I've revoked my commitment to  $p$  and accepted  $q$ , reasons for  $q$  are not reasons against  $p$ . Since there are good reasons for  $p$ , I'll now accept  $p$  again. We're now both committed to  $p$  and  $q$ , and neither of us are precluded from being entitled to either.

MADDY: Hmm . . . I guess so. Alright, great. I'm glad we were able to settle our disagreement.

NORM: No! You never *acknowledged* our disagreement! I challenged your claim to  $q$  on the grounds that  $p$ ,  $p$  is incompatible with  $q$ , and the grounds I have for  $p$  are better than the grounds you have for  $q$ . You claimed to agree with all of this. You said that you took  $p$  to be incompatible with  $q$ , and you agreed that my grounds for  $p$  were better than your grounds for  $q$ . You insisted, however, that, though  $p$  was incompatible with  $q$ ,  $q$  was not incompatible with  $p$ . But you never *really* took  $p$  to be incompatible with  $q$  at all.

MADDY: Sure I did. Isn't that why we had this whole discussion?

NORM: No, you never really took commitment to  $p$  to preclude entitlement to  $q$ . The whole time, you took it that someone who is committed to  $p$  *can* take on an additional commitment to  $q$ . You just took it that, in order to do this, they have to go through the detour of giving up  $p$ , accepting  $q$ , and then accepting  $p$  again. But this detour that you take it that someone who is committed to  $p$  must take in order to be committed to both  $p$

and  $q$  is *pointless*. It doesn't require one to take on any additional commitments. So someone who is committed to  $p$  can, without taking on any commitment other than  $q$ , come to be committed to both  $p$  and  $q$ . From a pragmatic perspective, your attitudes are exactly the same as those of someone who takes someone who is committed to  $p$  to be able to take on a commitment to  $q$  directly. So, I repeat, you don't *really* take  $p$  to be incompatible with  $q$ . Really taking  $p$  to be incompatible with  $q$  requires also taking  $q$  to be incompatible with  $p$ .

Norm reveals that, in this scenario, someone who is committed to  $p$  can, even by Maddy's lights, take on an additional commitment to  $q$ . They can do this simply by rejugling their commitments, revoking their acceptance of  $p$ , accepting  $q$ , and then accepting  $p$  again. Since it is always licit to revoke one's commitment in response to a challenge, one is always able to get to "the other side" of the incompatibility, and proceed from there.

The claim that Norm makes, concluding the above dialogue, is that asymmetric incompatibility is no real incompatibility at all. Incompatibility, understood pragmatically as I am understanding it here, must, at least potentially, be able to bear in *challenging* a claim. If  $p$  is incompatible with  $q$ , in the sense that commitment to  $p$  precludes entitlement to  $q$ , then an act of claiming  $q$  must be able to function to challenge to someone's commitment to  $p$ , even if we suppose that the converse does not hold. However, in this case, Maddy claims to take  $p$  to be incompatible with  $q$ , and yet, Maddy's claim of  $q$  is *not* able to function to challenge to Norm's commitment to  $p$ , since Norm can simply rejuggle his commitments and, maintaining a commitment to  $p$ , commit himself to  $q$  in a way that is perfectly licit by Maddy's lights. In this way, Maddy's attitude of taking  $p$  to be incompatible with  $q$  is utterly pragmatically inefficacious, and, for this reason, Norm claims that Maddy does not really have this attitude at all. We've considered the simplest case here, involving only two asymmetrically incompatible claims, but the point here extends straightforwardly to cases of more than two claims: asymmetric incompatibility is no incompatibility at all.

This argument bears a certain resemblance, at a high enough level of abstraction, to the Dutch Book arguments in probability theory. These arguments show that, if a practice of betting and bet-taking doesn't correspond to the probability axioms, then it will be defective in such a way that one player can essentially cheat the other out of all of their money, no matter what actually ends up happening with the events the bets are being placed on. This is a certain kind of "pragmatics-first" justification of the formal structure of the probability calculus, explaining why the calculus

must have the structure that it does by showing that any deviation from this structure would correspond to a defective betting practice, where one player is able to cheat the other out of all of their money no matter what, effectively trivializing the whole practice of betting. Likewise, we've offered a pragmatics-first justification of the formal structure of a certain semantic relation, incompatibility, explaining why it must have the symmetric structure that it does since a deviation from it would correspond to a defective practice of challenging, one in which a player is able to essentially "cheat" themselves out of a challenge by simply rejugling their commitments. The whole point of challenging is that it's supposed to make someone *choose*—to either give up the challenged commitment or defend it. If one doesn't take incompatibility relations to be symmetric, there's a way to "cheaply win" the game of giving and asking for reasons, no matter what one's actual reasons are, trivializing the whole practice.

Now, certain authors, such as Joyce (1998) and Pettigrew (2016), have argued that a *merely pragmatic* justification of the probability axioms of the sort provided by a Dutch Book argument is unsatisfactory, and offered alternative *epistemic* justifications of the probability axioms. One might worry that a similar concern of unsatisfactoriness, motivating an alternative justification, will apply here as well. However, I'll now argue that, though the justification of the symmetry of incompatibility offered here is "pragmatics-first" in that it is concerned with the practical consequences of treating incompatibility as asymmetric in the game of giving and asking for reasons, it's not right to say that it's a *merely pragmatic* justification. According to this argument, the symmetry of incompatibility relations, understood pragmatically as just those relations that underwrite challenges, is an essential structural feature of discursive practice as such. When we contextualize this argument in a pragmatist theory of propositional content of the sort developed by Brandom (1994), according to which propositional contents are conferred by discursive practices, this argument yields a result about propositional contents as such. It thereby concerns the structure of possible objects of knowledge as such, and so, though "pragmatic," is ultimately no less "epistemic" than, for instance, Kant's (1998) transcendental argument for the objective validity of the categories. Let me explain.

## 7 From Pragmatics Back to Semantics

I started this paper with an example involving worldly states of affairs: a car's being red is incompatible with its being blue. On Berto's conception of his proposed semantics for negation, the exclusion between these two



states is to be understood as a primitive *alethic modal* relation between their *contents*. I have claimed, by contrast, that only if we understand incompatibility, in the first instance, as a *normative* relation between discursive *acts* can we explain why it is that it must be a symmetric relation. I've now given an argument that incompatibility, normatively understood as a relation between acts, must be symmetric if it is to play its basic pragmatic role in a discursive practice, facilitating the challenging of a claim. This leaves us now with the following question: how does this get us back to our original explanandum? How have we thereby explained the symmetry of the *alethic modal* relation between the *contents*, rather than merely the *normative* relation between the *acts*?

To even start to answer this question, we must first carefully consider the notion of "content" that figures into the semantics at issue here. Consider, for instance, the state of affairs consisting of *a*'s being red. This state of affairs can be identified as the minimal truth-maker of the sentence "*a* is red," the state of affairs that must be included in any state of affairs in which this sentence holds. Accordingly, it's essential to the semantics that, even if *a* is actually red, and so a particular shade of red, the state of affairs consisting in the *a*'s being red is not to be understood in terms of *a*'s being just the shade of red that it actually is (a particular shade of crimson, say). A way to put this point is to say that the states of affairs that figure in the semantics are *general*, characterizing a range of possible ways that the world could be, rather than fully *particular*, characterizing just the way that the world actually is. This is why states of affairs can be understood in terms of their "informational content," even though this phrase is more commonly thought to apply to *propositions*, which are made true by the obtaining of states of affairs. The difference between a state of affairs and a proposition is a really a difference in *force* or *form*, rather than a difference in *content*. That is, for a pair consisting of a state of affairs and corresponding proposition, we have the same object/property (or objects/relation) complex, but the relation unifying object and property is distinct. In a state of affairs, the relation unifying object and property that of (possible) *instantiation*, whereas, in a proposition, the relation unifying object and property is that of (possible) *predication*. It is this difference in the unity of states of affairs and propositions that accounts for the fact that things of the former sort make true things of the latter sort. However, there is only one kind of "content" at issue here, possessed by both states of affairs and propositions: propositional content.

Now that we've clarified the notion of "content" at issue here, the key to answering the question is to contextualize the argument for the symmetry of incompatibility, understood in pragmatic terms, in the pragmatist the-

ory of content put forward by Brandom, according to which propositional contents are *conferred* by underlying practice of making claims and giving and asking for reasons for them. In *Making It Explicit*, Brandom (1994) aims to specify what practices a community must engage in—what members of a community must *do*—such that, engaging in those practices, members of that community can be counted as *saying* things—uttering sentences with propositional contents, made true by the obtaining of propositionally contentful states of affairs. The result of such an account of propositional contents is that there is a constitutive tie between what it is to be a propositional content and what it is to perform an act of undertaking an assertional commitment, which can be challenged and for which reasons can be given. There are different ways in which a pragmatist account of content along the lines proposed by Brandom can be determinately spelled out. On the most austere version of this picture, the propositional contents grasped by speakers of a language, which can be articulated in terms of the alethic modal relations they bear to one another, are simply tools for keeping track of how the social normative score changes through the making of various claims. For instance, a speaker keeps track of norms governing the making of the claim that *a* is red, which commits one to the claim that *a* is colored, precludes one from being entitled to the claim that *a* is blue, and so on through grasping the state of affairs consisting of *a*'s being red, whose obtaining necessitates *a*'s being colored, excludes *a*'s being blue, and so on. The grasp of such worldly contents, on such an account, is conferred upon speakers by their mastery of corresponding norms. If we think of contents as conferred by normative practices in this sort of way, then we can account for necessary structural features of contents by explicating the necessary structural features of any practice capable of conferring contents. This not only enables us to make sense of the argument here, but enables us to situate it in a wider class of “pragmatic transcendental arguments,” which can function to provide explanations of structural features of contents or concepts by explicating them in terms of corresponding structural features of any possible discursive practice.

In this connection, let me close by connecting the transcendental argument I've given here, drawing on Brandom, with another transcendental argument that has actually been made by Brandom, not for the symmetry of *incompatibility* but for the symmetry of *identity*. However absurd it might have seemed to think (at least at the start of this paper) that one could offer an argument for the symmetry of incompatibility, it might seem monumentally more absurd to think that one could offer an argument for the symmetry of identity. In effect, however, that is just what Brandom does in Chapter Six of *Making It Explicit*, though he never quite puts it

that way. Brandom identifies singular terms as the types of expressions that can be substituted *for* one another in different sentence frames. Accordingly, an identity claim such as “ $a = b$ ” expresses the goodness of a substitution inference from a claim of the form  $Fa$  (for any sentence frame  $Fx$ ) to a claim of the form  $Fb$ . The question of the symmetry of identity, then, becomes the question of whether these sorts of substitution inferences must be symmetric, or whether it is possible for there to be a discursive practice in which they are asymmetric? Brandom purports to show that no discursive practice that is capable of introducing conditionals could support expressions that are asymmetrically substitutable into sentence frames. Thus, insofar as a discursive practice is such that reasons can be made explicit, in the form of conditionals, the concept of identity on which members of that practice have a grip must be symmetric. I will not rehearse the details of Brandom’s argument here (see 1994, 376–384). The point is just to note the structure of the answer to the question of the symmetry of identity that Brandom’s argument can be seen as providing. The notion of identity is understood, in the first instance, *pragmatically*, in terms of what one *does* when one makes an identity claim—what kind of inferences one licenses. Brandom then shows that any practice in which these inferences are made asymmetrically could not actually function as a full-blown discursive practice.

This all becomes relevant to the present discussion when we consider how Berto takes incompatibility to be a *primitive* notion, and presumably takes this fact to mean that there is no way to give an account of its basic features such as its symmetry. Berto and Restall (2019) double down on this point, claiming that incompatibility belongs to a class of other “fundamental notions like reference, identity, necessity, or negation,” (2019: 4). The basic structural features of these notions, it seems, can only be *explicitated*; they can’t be *explained*. As Berto says, “Explanations stop when we reach concepts that cannot be defined in terms of other concepts, but only illustrated by way of example,” (2015, 768). But there is another kind of explanation of a concept that is distinct from a semantic *reduction* of that concept, and that is a pragmatic *deduction* of that concept. I hope to have shown, by way of example, what such a pragmatic deduction can look like.<sup>28</sup>

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## References

- [1] Aristotle (2016) *Metaphysics*, trans. C.D.C. Reeve. Indianapolis: Hackett.
- [2] Berto, F (2008) “*Αδυνατον* and material exclusion”, *Australasian Journal of Philosophy* 86/2:165-190.
- [3] Berto, F. (2015) “A Modality Called ‘Negation’”, *Mind* 124/495: 761-793.
- [4] Berto, F and G. Restall (2019) “Negation on the Australian Plan”, *Journal of Philosophical Logic* 48/6: 1119-1144.
- [5] Berto, F. and M. Jago (2019) *Impossible Worlds*. Oxford: Oxford University Press.
- [6] Brandom, R. (1994) *Making It Explicit*. Cambridge, MA: Harvard University Press.
- [7] Brandom, R. (2008) *Between Saying and Doing*. Oxford: Oxford University Press.
- [8] Brandom, R. (2018) “From Logical Expressivism to Expressivists Logics: Sketch of a Program and Some Implementations,” in O. Beran, V. Kolman, and L. Koren (eds.) *From Rules to Meanings: New Essays on Inferentialism*, 151-164. New York: Routledge.
- [9] Brandom, R. (2019) *A Spirit of Trust*. Cambridge, MA: Harvard University Press.
- [10] De, M. and H. Omori (2018) “There is More to Negation than Modality”, *Journal of Philosophical Logic* 47: 281-299.
- [11] Dunn, M. (1993) “Star and Perp: Two Treatments of Negation”, in J. Tomberli (ed.) *Philosophical Perspectives 7: Language and Logic*, 331-357. Atascadero CA: Ridgeview.
- [12] Dunn, M. (1996) “Generalized Ortho Negation”, in H. Wansing (ed.) *Negation: A Notion in Focus*, 3-26. Berlin: de Gruyter.
- [13] Dunn, M. (1999) “A Comparative Study of Various Model-Theoretic Treatments of Negation: A History of Formal Negation,” in D. M. Gabbay and H. Wansing (eds.) *What Is Negation?*, 23-51. Dordrecht: Springer.

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- [14] Fine, K. (2017) "Truthmaker Semantics", in B. Hale, C. Wright, and A. Miller (eds.) *A Companion to the Philosophy of Language* 2nd edn., 556-577. John Wiley & Sons.
- [15] von Fintel, K. (2001) "Counterfactuals in Dynamic contexts", in M. J. Kenstowicz (ed.) *Ken Hale: A Life in Language*, 123-152. Cambridge, MA: MIT University Press.
- [16] Francez, N. (2019) "Another Plan for Negation", *Australasian Journal of Logic* 16/5: 159-176.
- [17] Gillies, A. (2007) "Counterfactual Scorekeeping", *Linguistics and Philosophy* 30: 329-360.
- [18] Golan, R. (2023) "A Unified Interpretation of the Semantics for Relevance Logic", *Mind*.
- [19] Hartonas, C and J. M. Dunn (1993) "Duality Theorems for Partial Orders, Semilattices, Galois Connections and Lattices", *Indiana University Logic Group Preprint no. IULG-93-26*.
- [20] Hazen, A. P. (1995) "Is Even Minimal Negation Constructive?" *Analysis* 55/2: 105-107.
- [21] Hlobil, U. (2016) "A Nonmonotonic Sequent Calculus for Inferentialist Expressivists." In P. Arazim and M. Dančák (eds.) *The Logica Yearbook 2015*, 87-105. London: College Publications.
- [22] Hlobil, U. (2023) "The Laws of Thought and the Laws of Truth as Two Sides of One Coin", *Journal of Philosophical Logic* 52:313-343.
- [23] Joyce, J. M. (1998) "A Nonpragmatic Vindication of Probabilism", *Philosophy of Science* 65/4: 575-603.
- [24] Kant, I. (1998) *The Critique of Pure Reason*, trans. and ed. by P. Guyer and A. Wood. Cambridge: Cambridge University Press.
- [25] Kaplan, D. (2021) *Substructural Content*. Ph.D. Dissertation, University of Pittsburgh.
- [26] Lennertz, B. (2019) "Might-Beliefs and Asymmetric Disagreement", *Synthese* 196: 4775-4805.
- [27] Lewis, D. (1968) "Counterpart Theory and Quantified Modal Logic", *Journal of Philosophical Logic* 65/5: 113-126.

- [28] Lewis, D. (1986) *On the Plurality of Worlds*. Wiley Blackwell.
- [29] Meyer, R. and Martin, E. (1986) "Logic on the Australian Plan", *Journal of Philosophical Logic* 15: 305-332.
- [30] Nickel, B. (2013) "Dynamics, Brandom-Style", *Philosophical Studies* 162/2: 333-354.
- [31] Omori, H. and De, M. (2022) "Shrieking, Shrugging, and the Australian Plan", *Notre Dame Journal of Formal Logic* 63: 137-141.
- [32] Pettigrew, R. (2016) *Accuracy and the Laws of Credence*. Oxford: Oxford University Press.
- [33] Porello, D. (2012) "Incompatibility Semantics from Agreement", *Philosophia* 40: 99-119.
- [34] Priest, G. (1979) "The Logic of Paradox", *Journal of Philosophical Logic* 8/1: 219-241.
- [35] Priest, G. (1997) "Sylvan's Box: A Short Story and Ten Morals", *Notre Dame Journal of Formal Logic* 38/4: 573-582.
- [36] Restall, G. (1999) "Negation in Relevant Logics (How I stopped worrying and learned to love the Routley Star)", in D. M. Gabbay and H. Wansing (eds.) *What Is Negation?*, 53-76. Dordrecht: Springer.
- [37] Restall, G. (2000) "Defining Double Negation Elimination", *Logic Journal of the IGPL* 8/6: 853-860.
- [38] Restall, G. (2005) "Multiple Conclusions", in P. Hájek, L. Valdés-Villanueva and D. Westerstahl (eds.) *Logic, Methodology and Philosophy of Science*. College Publications.
- [39] Sellars, W. (1953) "Inference and Meaning", *Mind* 62/247: 313-338.
- [40] Stalnaker, R. (1984) *Inquiry*. Cambridge, MA: MIT Press.
- [41] Wanderer, J. (2010) "Brandom's Challenges." In J. Wanderer and B. Weiss (eds.) *Reading Brandom: On Making It Explicit*, 96-114. New York: Routledge.
- [42] Willer, M. (2017) "Lessons from Sobel Sequences", *Semantics and Pragmatics* 10/4: 1-57.